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Short communication

Transient-boundary voltage method for measurement of equivalent circuit components of rechargeable batteries

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ABSTRACT

A method is presented for measuring the equivalent circuit components of rechargeable batteries. The temporal discharge–rest–charge–rest sequence of a rechargeable battery is described, using the principles of transient circuit analysis, to derive equations for the battery voltage as a function of time during voltage transients and at the boundaries at transitions between transient phases. The equations lead to a new measurement method for battery characterization. The equivalent circuit of the battery is described as an ideal voltage source in series with a resistor and the parallel combination of a resistor and a capacitor. The battery model uses different values of resistance and capacitance, in the parallel combination, during the different phases of the discharge–rest–charge–rest sequence. The method is used to measure the circuit parameters of a nickel–cadmium battery.

1. Introduction

The engineering design and performance evaluation of battery dependent systems require an electrical equivalent circuit for the battery. The equations for a method to measure the equivalent circuit components of rechargeable batteries are derived in this paper.

The most basic circuit of the battery is a voltage source in series with the internal resistance. The electrochemical processes of rechargeable batteries [1,2] lead to a second, and more precise, equivalent circuit comprising a voltage source in series with a resistor and a parallel combination of a resistor and a capacitor [3–7]; the voltage–resistance–capacitance (VRC) model. The latter is used to model the battery under conditions of constant state of charge and constant temperature. This model is valid for discharge times of the order of seconds. Thirdly, battery models that apply to longer discharge times take into account changes in battery voltage and resistance, battery state of charge and electrolyte temperature [8–10].

Different measurements techniques, which are based on the above models, have been used to determine equivalent circuit parameters of secondary batteries. These methods include the current-voltage method for the measurement of internal resistance [11], the Laplace transform method for cell impedance measurements [12] and the impedance spectrum method [13,14]. A battery measurement method based on transient analysis has not been fully developed.

The basic VRC model was used in the present work to describe the processes of discharge–rest–charge–rest sequence of a rechargeable battery with different values for the resistor and capacitor, in the parallel combination of R and C, for the charge and discharge phases and for the intervening rest periods. Equations for the temporal variation of the battery voltage during transients, including boundaries at transitions between transients, are derived in this paper. The derivation is based on the principles of transient circuit analysis [15]. The equations lead to a new method of measurement of the equivalent circuit parameters of a secondary battery. The technique requires the measurement of the abrupt changes in the battery voltage at boundaries between transients in the discharge–rest–charge–rest sequence. The advantage of the technique is the ease of conducting the experiment and the use of readily available, inexpensive equipment; it takes one run to complete the experiment using a voltage source in the charging phase and a voltmeter to measure the battery voltage throughout the run. The method is used here to determine the circuit parameters of a nickel–cadmium battery.

2. Theory of method

2.1. Battery discharge–rest equations

Transients are involved during the charging and discharging of batteries. Abrupt changes are recorded in the voltage across the terminals of the battery as the switch is closed or opened at the start of these transients. These boundary value changes in voltage as well as the intervening transients are derived here. The equivalent circuit of the battery appears to the left of terminals b_1 and b_2 in

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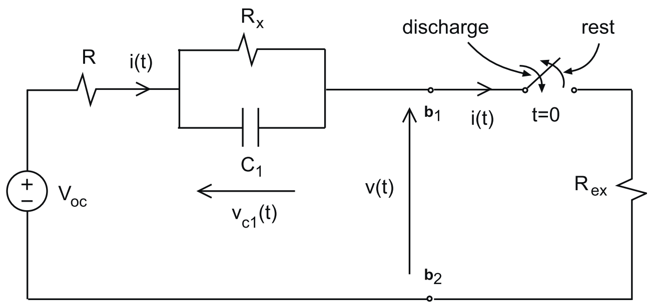

Fig. 1. The discharge phase. The circuit for the discharge and rest periods of the battery. The charged battery starts discharging current through the external load resistor R_{ex} at time $t = 0$. $R_x = R_1$ for the discharge period and $R_x = R_2$ for the rest period.

Fig. 1 where $v(t)$ is the battery voltage. It is a series combination of the open circuit battery voltage V_{oc}, a resistor, and the equivalent of the parallel combination of a resistor and a capacitor.

Fig. 1 shows the battery equivalent circuit before and during the discharge phase and in the rest period that follows the discharge phase. $R_x = R_1$ for the discharge period and $R_x = R_2$ for the rest period.

Consider first a fully charged battery with the switch in Fig. 1 open. The switch is then closed at time $t = 0$ and the battery starts to discharge a current through the external load resistance R_{ex}. The battery voltage eventually stabilizes to its steady state value $v(\infty)$. The aim is to find the equation for the variation in the battery voltage $v(t)$ first during the discharge period.

The capacitor C_1 carries no charge for $t < 0$ and so the voltage across the capacitor just before and just after the switch is closed, at $t = 0^-$ and 0^+, respectively, is $v_{C_1}(0^-) = v_{C_1}(0^+) = 0$. The capacitor voltage during the discharge transient ($t > 0$) is

$$v_{C_1}(t) = \frac{R_1 V_{oc}}{R + R_1 + R_{ex}} \left[1 - \exp\left(\frac{-t}{R_{eq1} C_1} \right) \right] \quad (1)$$

where

$$R_{eq1} = \frac{R_1 (R + R_{ex})}{R + R_1 + R_{ex}} \quad (2)$$

The battery voltage during the transient is

$$v(t) = R_{ex} \left[\frac{v_{C_1}(t)}{R_1} + C_1 \frac{dv_{C_1}(t)}{dt} \right] \quad (3)$$

$$v(t) = \frac{R_{ex} V_{oc}}{R + R_1 + R_{ex}} \left\{ 1 + \frac{R_1}{R + R_{ex}} \exp\left(\frac{-t}{R_{eq1} C_1} \right) \right\} \quad (4)$$

Also, as the switch is closed at the start of the discharge period ($t = 0$), the battery voltage changes instantaneously from $v(0^-) = V_{oc}$ to

$$v(0^+) = \frac{R_{ex} V_{oc}}{R + R_{ex}} \quad (5)$$

During the discharge transient the battery and capacitor voltages tend to $v(\infty)$ and $v_{C_1}(\infty)$, respectively, where

$$v(\infty) = \frac{R_{ex} V_{oc}}{R + R_1 + R_{ex}} \quad (6)$$

$$v_{C_1}(\infty) = \frac{R_1 V_{oc}}{R + R_1 + R_{ex}} \quad (7)$$

Considering the rest period, which starts when the switch in Fig. 1 is opened at a new time origin $t = 0$, the battery terminals b_1 and b_2 will be on open circuit and the voltage across them will undergo a sudden change, and then display a transient as $v(t)$ increases and tends to V_{oc}. The initial voltage across the capacitor will equal the final voltage across this capacitor at the end of the discharge phase and is given by Eq. (7). The resistor R_1 for the discharge circuit, has been replaced by R_2 in the resting one, because the two transients do not necessarily follow the same time constant. For the rest period the voltage across the capacitor opposes V_{oc}. As the capacitor discharges through R_2, with increasing time, however, the battery voltage $v(t)$ increases and tends to V_{oc}. The battery voltage during the rest transient is

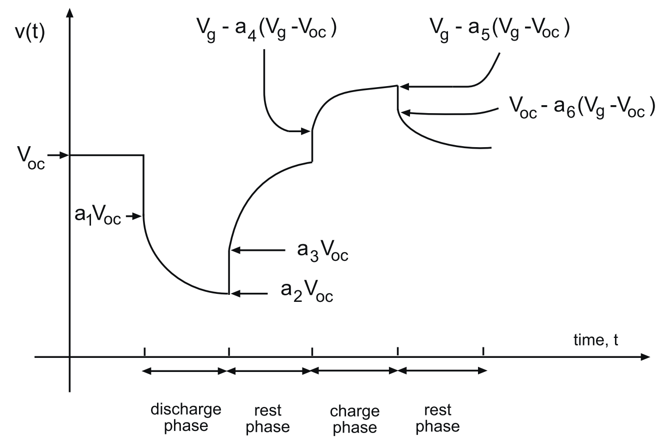

Fig. 2. The battery voltage. The battery terminal voltage as a function of time during the discharge and rest phases, followed by the charge and rest phases.

$$v(t) = V_{oc} - v_{C_1}(t) \quad (8)$$

$$v(t) = V_{oc} - \frac{R_1 V_{oc}}{R + R_1 + R_{ex}} \exp\left(\frac{-t}{R_2 C_1} \right) \quad (9)$$

Also, as the switch is opened at the start of the rest period, the battery voltage changes instantaneously from $v(0^-)$, given by Eq. (6), to

$$v(0^+) = \left(\frac{R + R_{ex}}{R + R_1 + R_{ex}} \right) V_{oc} \quad (10)$$

The battery voltage during the rest transient tends to

$$v(\infty) = V_{oc} \quad (11)$$

The temporal variation of the voltage across the battery terminals during the discharge phase and the subsequent rest period appears in Fig. 2.

In summary, and in accordance with the constants used in Fig. 2, the voltage across the battery terminals decreases abruptly from V_{oc} to $a_1 V_{oc}$ at the start of discharge, where

$$a_1 V_{oc} = \frac{R_{ex}}{R + R_{ex}} V_{oc} \quad (12)$$

The battery voltage during the discharge transient tends to $a_2 V_{oc}$, where

$$a_2 V_{oc} = \frac{R_{ex}}{R + R_1 + R_{ex}} V_{oc} \quad (13)$$

As the battery terminals are opened at the start of the subsequent rest period, the battery voltage changes abruptly from $a_2 V_{oc}$ to $a_3 V_{oc}$ where

$$a_3 V_{oc} = \frac{R + R_{ex}}{R + R_1 + R_{ex}} V_{oc} \quad (14)$$

2.2. Battery charge–rest equations

Fig. 3 shows the battery circuit just before and during the charge phase. The charge phase is followed by a rest period. The same battery model applies as in the discharge phase but now with different values for the components in the parallel circuit. The capacitor

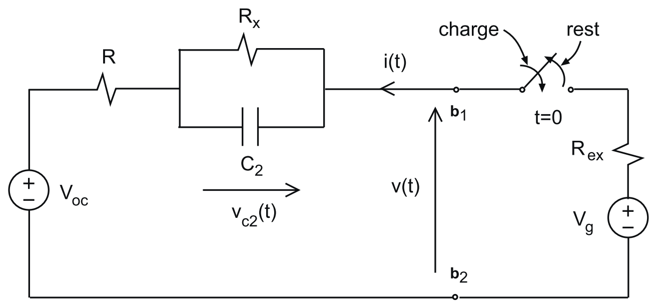

Fig. 3. The charge phase. The circuit for the battery in the charging phase followed by the resting period. V_g is the external charging voltage. $V_g > V_{oc}$. $R_x = R_3$ for the charging period and $R_x = R_4$ for the rest period.

value is C_2 for both the charge and the rest periods, while R_3 and R_4 are the resistors in parallel with the capacitor in the charge and rest periods, respectively. R stays the same as in the discharge–rest case.

Since $V_g > V_{oc}$ the voltage source V_g acts as a generator. The current $i(t)$ flows into terminal b_1 and ultimately into the positive terminal of V_{oc} as a charging current to the battery. The source V_{oc} now acts as the energy absorber. The aim is to find the equation for the variation in the battery voltage $v(t)$, first during the charge, and then for the subsequent rest phase.

Considering the charging phase first, the capacitor voltage just before and just after the switch is closed, at $t = 0^-$ and 0^+, respectively, at the start of the new time origin at $t = 0$, is $v_{C_2}(0^-) = v_{C_2}(0^+) = 0$. The voltage across the capacitor during the charging transient for $t > 0$ is then given by

$$v_{C_2}(t) = \frac{R_3}{R + R_3 + R_{ex}}(V_g - V_{oc})\left\{1 - \exp\left[\frac{-t}{R_{eq2}C_2}\right]\right\} \tag{15}$$

where R_{eq2} is

$$R_{eq2} = \frac{R_3(R + R_{ex})}{R + R_3 + R_{ex}} \tag{16}$$

The battery voltage is

$$v(t) = V_g - R_{ex}\left(\frac{v_{C_2}(t)}{R_3} + C_2\frac{dv_{C_2}(t)}{dt}\right) \tag{17}$$

$$v(t) = V_g - \frac{R_{ex}(V_g - V_{oc})}{R + R_3 + R_{ex}}\left\{1 + \frac{R_3}{R + R_{ex}}\exp\left[\frac{-t}{R_{eq2}C_2}\right]\right\} \tag{18}$$

Also, as the switch is closed at the start of charging ($t = 0$), the battery voltage changes instantaneously from $v(0^-) = V_{oc}$ to

$$v(0^+) = V_g - \frac{R_{ex}}{R + R_{ex}}(V_g - V_{oc}) \tag{19}$$

The battery voltage during the charging transient tends to

$$v(\infty) = V_g - \frac{R_{ex}}{R + R_3 + R_{ex}}(V_g - V_{oc}) \tag{20}$$

Considering the resting phase that follows charging, when now the battery terminals b_1 and b_2 are on open circuit, the battery voltage will undergo a sudden change and then display a transient as $v(t)$ decreases and tends to V_{oc}. Starting the rest phase at a new time origin $t = 0$, the initial voltage across the capacitor will equal the final voltage across it at the end of the charge phase, and is, from Eq. (15), given by

$$v_{C_2}(0^+) = \frac{R_3}{R + R_3 + R_{ex}}(V_g - V_{oc}) \tag{21}$$

The voltage across C_2 aids V_{oc} and as a result, for the early part of the rest period $v(t) > V_{oc}$. As time increases, however, and the capacitor discharges through R_4, the battery voltage $v(t)$ decreases, and tends to V_{oc}. The battery voltage during the resting transient is

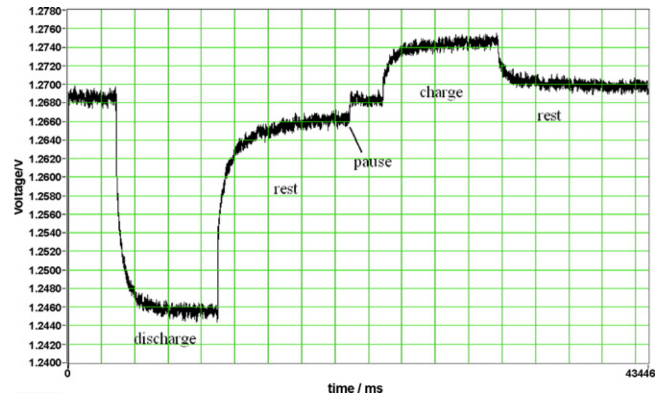

Fig. 4. The characteristics of a rechargeable battery. The discharge–rest–charge–rest characteristics of a NiCad battery.

$$v(t) = V_{oc} + v_{C_2}(t) \tag{22}$$

$$v(t) = V_{oc} + \frac{R_3}{R + R_3 + R_{ex}}(V_g - V_{oc})\exp\left[\frac{-t}{R_4C_2}\right] \tag{23}$$

Also, as the switch is opened at the start of the rest period the battery voltage changes abruptly from $v(0^-)$, given by Eq. (20), to

$$v(0^+) = V_{oc} + \frac{R_3}{R + R_3 + R_{ex}}(V_g - V_{oc}) \tag{24}$$

At the end of the rest transient the battery voltage tends to

$$v(\infty) = V_{oc} \tag{25}$$

The voltage variation across the battery terminals during the charge phase and the subsequent rest period, as a function of time, appears in Fig. 2. Summarizing the boundary conditions, and in accordance with the constants used in Fig. 2, the battery voltage increases instantaneously from V_{oc} to

$$V_g - a_4(V_g - V_{oc}) = V_g - \frac{R_{ex}}{R + R_{ex}}(V_g - V_{oc}) \tag{26}$$

as charging starts.

During the charging transient the battery voltage tends to

$$V_g - a_5(V_g - V_{oc}) = V_g - \frac{R_{ex}}{R + R_3 + R_{ex}}(V_g - V_{oc}) \tag{27}$$

As the switch is opened at the start of the rest period, the battery voltage decreases instantaneously to

$$V_{oc} - a_6(V_g - V_{oc}) = V_{oc} + \frac{R_3}{R + R_3 + R_{ex}}(V_g - V_{oc}) \tag{28}$$

3. Results and discussion

Fig. 4 shows the discharge–rest–charge–rest characteristics of a 1.2 V, 1300 mAh nickel–cadmium battery where the battery voltage is plotted as a function of time. The battery was fully charged before the experiment. Signal acquisition was interrupted for 1.75 h during the rest phase that followed the discharge one, at the point marked "pause", to allow the battery to recover and reach its initial open circuit voltage. The signal acquisition was then resumed to proceed with the recording of the charge–rest phases. The charging voltage V_g was 1.634 V. The external load resistor R_{ex} was 10 Ω.

Gradients of the battery voltage at the start of various transients were used, in addition to the equations derived earlier, for the calculations of the battery circuit parameters. The respective gradients

Table 1
Component values of the equivalent circuit of the battery. The number that appears to the right of every value refers to the equation that was used to calculate the result.

Component	Value	Equation	Component	Value	Equation
R	0.071 Ω	12	R	0.055 Ω	26
R_1	0.121 Ω	13	R_3	0.140 Ω	27
R_1	0.120 Ω	14	R_3	0.136 Ω	28
R_2	0.250 Ω	30	R_4	0.091 Ω	32
C_1	3.427 ± 0.117 F	29	C_2	9.176 ± 2.364 F	31

at the start of the discharge phase and at the start of the subsequent rest period are given by

$$\frac{dv}{dt} = -\frac{R_{ex}V_{oc}}{(R+R_{ex})^2C_1} \tag{29}$$

$$\frac{dv}{dt} = \frac{R_1V_{oc}}{(R+R_1+R_{ex})R_2C_1} \tag{30}$$

The respective gradients at the start of the charge phase and at the start of the subsequent rest period are given by

$$\frac{dv}{dt} = \frac{R_{ex}(V_g-V_{oc})}{(R+R_{ex})^2C_2} \tag{31}$$

$$\frac{dv}{dt} = -\frac{R_3(V_g-V_{oc})}{(R+R_3+R_{ex})R_4C_2} \tag{32}$$

The component values of the equivalent circuit of the battery are given in Table 1. The values of R were calculated separately for the discharge and charge phases and used in subsequent calculations for respective discharge–rest and charge–rest phases. There is very good agreement between the two values for R_1 calculated using Eqs. (13) and (14). Also, the same is true for R_3 calculated from Eqs. (27) and (28). C_1 and C_2 were calculated from Eqs. (29) and (31), respectively, and so rely on the gradient at the start of the discharge phase and that at the start of the charge one. There is an uncertainty, and a corresponding margin of error, in determining the gradient. The calculated values of C_1 and C_2, based on the maximum and minimum possible gradients, determined by the author for the start of each transient, are given in Table 1. It can be seen that the method provides the value of C_1 with greater reliability than C_2. The calculated equivalent circuit parameters provide a precise circuit model for the battery.

4. Conclusions

The derivation of the voltage variation with time of a rechargeable battery during charging and discharging and in the boundary regions between these phases has led to equations that can be used in a measurement method to determine the component values of the equivalent circuit of rechargeable batteries. The use of the measurement method in battery characterization was illustrated in relation to a nickel–cadmium battery. The resulting equivalent circuit parameters provide a precise circuit model for the battery for use in the design and study of battery powered circuits. The technique can be used in future battery studies to determine battery circuit parameters under different states of charge and at different battery temperatures.

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